Assignment 2.

This homework is due *Tuesday*, September 22 (because I will cover some stuff related to Problems 5–7 on Tuesday, Sep 15).

There are total 22 points in this assignment. 17 points is considered 100%. If you go over 16 points, you will get over 100% for this homework and it will count towards your course grade (but not over 115%).

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper and *credit your collaborators*. Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers sectionss 1.2, 1.3, 2.1 in Bartle–Sherbert.

(1) [2pt] (Paragraph 1.2.4g) Find a mistake in the following (erroneous!) induction argument:
Claim: If $n \in \mathbb{N}$ and if the maximum of the natural numbers p, q is n, then $p = q$.
"Proof." Proof by induction in n. Evidently, for $n = 1$ claim is true since in such case, $p = 1$ and $q = 1$.
Suppose, the claim holds for some $n \in \mathbb{N}$. Prove that then it also holds for $n+1$. Suppose maximum of p and q is $n+1$. Then maximum of $p-1$ and $q-1$ is $(n+1)-1 = n$. By induction hypothesis, $p-1 = q-1$, therefore $p = q$. Thus, the claim holds for $n+1$ and, by induction principle, for all natural numbers.

(2) [2pt] (1.3.5) Exhibit (define explicitly) a bijection from N to the set of all odd integers greater than 13.

(3) Exhibit (define explicitly) a bijection between
 (a) [2pt] $\mathbb{Z}$ and $\mathbb{Z} \setminus \{0\}$,
 (b) [3pt, optional[1]] $\mathbb{Q}$ and $\mathbb{Q} \setminus \{0\}$.

(4) [3pt] (1.3.12) Prove that the collection $\mathcal{F}(\mathbb{N})$ of all *finite* subsets of $\mathbb{N}$ is countable.

(5) [2pt] Prove that there does not exist a rational number r such that $r^2 = 3$.

(6) [4pt]
 (a) ($\sim$2.1.8a) Let x, y be rational numbers. Prove that $xy, x-y$ are rational numbers.
 (b) (2.1.8b) Let x be a rational number, y an irrational number. Prove that $x+y$ is irrational. Prove that if, additionally, $x \neq 0$, then xy is irrational.
 (c) Let x, y be irrational numbers. Is it true that xy is always irrational? Is it true that xy is always rational?

(7) [4pt] (2.1.9) Let $K = \{s + t\sqrt{2} \mid s, t \in \mathbb{Q}\}$. Show that K satisfies the following:
 (a) If $x_1, x_2 \in K$ then $x_1 + x_2 \in K$ and $x_1 x_2 \in K$.
 (b) If $x \neq 0$ and $x \in K$ then $1/x \in K$. (*Hint:* Get rid of irrationality in the denominator..)
 Comment. In other words, K is a subfield of $\mathbb{R}$.

[1]That is, not included in the denominator of the grade.